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Tadpole versus anomaly cancellation in $D = 4, 6$ compact IIB orientifolds

Gerardo Aldazabal* **and Daniel Badagnani**

Instituto Balseiro, Universidad Nacional de Cuyo,
8400 S.C. de Bariloche, Argentina
E-mail: aldazaba@cab.cnea.gov.ar, daniel@cabtep2.cnea.edu.ar

Luis E. Ibáñez

Departamento de Física Teórica C-XI and Instituto de Física Teórica C-XVI,
Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
E-mail: ibanez@madriz1.ft.uam.es

Angel M. Uranga

Institute for advanced Study, Olden Lane,
Princeton NJ 08540, USA
E-mail: uranga@ias.edu

Abstract: It is often stated in the literature concerning $D = 4, 6$ compact type IIB orientifolds that tadpole cancellation conditions (i) uniquely fix the gauge group (up to Wilson lines and/or moving of branes) and (ii) are equivalent to gauge anomaly cancellation. We study the relationship between tadpole and anomaly cancellation conditions and qualify both statements. In general the tadpole cancellation conditions imply gauge anomaly cancellation but are stronger than the latter conditions in $D = 4$, $N = 1$ orientifolds. We also find that tadpole cancellation conditions in $\mathbb{Z}_N$ $D = 4, 6$ compact orientifolds do not completely fix the gauge group and we provide new solutions different from those previously reported in the literature.

Keywords: D-branes, Superstring Vacua, Anomalies in Field and String Theories.

*CNEA, Centro Atómico Bariloche, 8400 S.C. de Bariloche, and CONICET, Argentina.

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1. Introduction

It is a well-known fact that anomaly cancellation in SO(32) type I string may be understood as a direct consequence of the cancellation of tadpoles of Ramond-Ramond fields. In fact the implication runs in both directions and anomaly cancellation implies also tadpole cancellation. This is not so surprising since in $D = 10$ the anomaly cancellation constraints fix almost uniquely the massless particle content of the theory.

In type I vacua in lower dimensions, like in type IIB $D = 4, 6$ orientifolds, tadpole cancellation constraints do also imply anomaly cancellation. An interesting question arises regarding the extent up to which anomaly cancellation and tadpole cancellation are still equivalent in lower dimensions. One of the purposes of this paper is to address this question. We concentrate in the study of $D = 4$, $N = 1$ type IIB compact orientifolds, although analogous results are obtained in $D = 6$. We find that tadpole cancellation is in general a more stringent constraint than anomaly cancellation in the case of $D = 4$ orientifolds. In order to show this, we rewrite the anomaly cancellation conditions in terms of traces of Chan-Paton twist matrices acting on 9-branes and/or 5-branes. In this way, it is shown that in general only a subset of the complete tadpole cancellation conditions is recovered. Thus, there are certain tadpole cancellation constraints which are actually not required for anomaly cancellation. Which those are depends on the structure of the twist group of the orientifold and also on the simultaneous presence or not of 9-branes and 5-branes.[1]

This procedure allows a broad scan for solutions of tadpole cancellation conditions in $D = 4, 6$ $N = 1$, type IIB orientifolds. In particular, we find that there are certain $D = 4$ and $D = 6$ orientifolds which admit many more solutions than previously reported in the literature.

The relationship between tadpole cancellation and anomaly cancellation was considered in ref. [2][2] in the context of $D = 4$, $N = 1$ gauge theories on the worldvolume of D3-branes sitting at $\mathbb{Z}_N \times \mathbb{Z}_M$ singularities.[3] In that reference an equivalence between tadpole and anomaly cancellation conditions was found. Those models differ from the class we consider in several important respects. In particular they are non-compact models (the six dimensions transverse to the D3-branes are not compactified) and in addition they only contain D3-branes.

In this paper, we consider *compact* orientifolds and this fact makes the following important difference. Tadpoles are sources for the RR potentials of the theory. In non-compact models some of the twisted RR fields can propagate on non-compact directions and carry the RR-flux off to infinity. The models are consistent without imposing cancellation of the corresponding twisted tadpoles. In the compact models we are to consider, the RR-flux cannot escape to infinity but is trapped in the compact space. Thus additional constraints may appear.

A second difference is that we consider the simultaneous presence of two types of D-branes, 9-branes, and 5-branes. This is equivalent by T-duality to considering both 3-branes and 7-branes in the context of [2]. However, in this reference the emphasis is in the gauge theory on the worldvolume of the D3-branes, while the D7-branes are considered non dynamical. This is justified since the D7-branes have more non-compact dimensions than the D3-branes. In our compact models, however, D7 branes wrap compact direction and yield truly four-dimensional fields. Thus the cancellation of anomalies from gauge groups living on the D7-branes lead to additional constraints, not present in the non-compact models. The considerations in this and the preceding paragraph explain the difference between our conclusions and those for non-compact models in [2].

The outline of the paper is as follows. In the next section, we review some facts about compact $D = 4$, $N = 1$ type IIB orientifolds and establish the notation needed for the remaining sections. In section 3.1, we study the compact orientifolds without even order generators, i.e. $\mathbb{Z}_3$, $\mathbb{Z}_7$, and $\mathbb{Z}_3 \times \mathbb{Z}_3$ orientifolds. In this case the models

[1] We are referring here to chiral $D = 4$ orientifolds. Non-chiral ones like the $\mathbb{Z}_2 \times \mathbb{Z}_2$ model of ref. [1] obviously do not get any constraint from anomaly cancellation.

[2] See also [3, 4, 5] for analogous results in six-dimensional orientifolds.

[3] $N = 1$ supersymmetric theories form D3-branes at orbifold singularities have been studied in [6, 7, 8, 9, 10]. The inclusion of orientifold projections has been discussed in [11, 12, 13, 14].

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have only D9-branes and tadpole conditions uniquely fix the gauge group. However in the $\mathbb{Z}_3 \times \mathbb{Z}_3$ case it is shown how tadpole cancellation conditions are stronger than anomaly cancellation conditions. In section 3.2 we study the compact orientifolds with even order twists. In this case both D9-branes and D5-branes are present. The $\mathbb{Z}_4$, $\mathbb{Z}_8$, $\mathbb{Z}_8{}'$, and $\mathbb{Z}_{12}{}'$ orientifolds with standard orientifold projection are shown to be necessarily anomalous, in agreement with the results of ref. [18] in which they were shown to have non-vanishing tadpoles. We present other examples (the $\mathbb{Z}_{12}$ orientifold) in which tadpole conditions are explicitly shown to be stronger than anomaly cancellation ones. New solutions for the tadpole conditions are shown for the $\mathbb{Z}_6$ and $\mathbb{Z}_{12}$ orientifolds leading to a variety of gauge groups previously overlooked in the literature. We study the cancellation of $U(1)$ anomalies in section 4. We find that cancellation of non-Abelian anomalies guarantees the cancellation of Abelian anomalies. In addition it is shown that in the $D = 4$ compact orientifold case cancellation of non-Abelian anomalies also implies the cancellation of mixed $U(1)$-gravitational anomalies. Section 5 is left for some final comments and conclusions. In particular we compare our results to those found for non-compact orientifolds and discuss the origin of the non-equivalence of tadpole/anomaly conditions. We also briefly discuss the equivalent results for $D = 6$, $N = 1$ compact IIB orientifolds.

2. $D = 4$, $N = 1$, type IIB orientifolds

In this section, we summarize the basic ingredients [15, 16, 17, 18] and notation needed in the construction of $D = 4, N = 1$ orientifold. The reader is referred to [18] for further details.

In a type IIB orientifold, the toroidally compactified theory is divided out by the joint action of a discrete symmetry group G_1, (like $\mathbb{Z}_N$ or $\mathbb{Z}_N \times \mathbb{Z}_M$) together with a world sheet parity operation Ω, exchanging left and right movers. Ω action can be accompanied by extra operations thus leading to generic orientifold group $G_1 + \Omega G_2$ with $\Omega h\, \Omega h' \in G_1$ for $h, h' \in G_2$.

In this article, we will refer to the cases $G_1 = G_2$ and $G_1 = \mathbb{Z}_N$ or $G_1 = \mathbb{Z}_N \times \mathbb{Z}_M$ and such that $D = 4$ $N = 1$ theories are obtained, when the twist Ω is performed on type IIB compactified on T^6/G_1. The allowed orbifold groups, acting crystalographically on T^6 leading to $N = 1$ unbroken supersymmetry were classified in [19]. The list, with corresponding twist vector eigenvalues $v = (v_1, v_2, v_3)$ associated to the $\mathbb{Z}_N$ orbifold twist θ is given in table 1.

Orientifolding closed type IIB string introduces a Klein-bottle unoriented world-sheet. Amplitudes on such a surface contain tadpole divergences. In order to eliminate such unphysical divergences Dp-branes must be generically introduced. In this way, divergences occurring in the open string sector cancel up the closed sector ones and produce a consistent theory. Tadpole cancellation is interpreted as cancellation of the charge carried by RR form potentials. For $\mathbb{Z}_N$, with N odd, only D9-branes

$\mathbb{Z}_3$	$\frac{1}{3}(1,1,-2)$	$\mathbb{Z}_6{}'$	$\frac{1}{6}(1,-3,2)$	$\mathbb{Z}_8{}'$	$\frac{1}{8}(1,-3,2)$
$\mathbb{Z}_4$	$\frac{1}{4}(1,1,-2)$	$\mathbb{Z}_7$	$\frac{1}{7}(1,2,-3)$	$\mathbb{Z}_{12}$	$\frac{1}{12}(1,-5,4)$
$\mathbb{Z}_6$	$\frac{1}{6}(1,1,-2)$	$\mathbb{Z}_8$	$\frac{1}{8}(1,3,-4)$	$\mathbb{Z}_{12}{}'$	$\frac{1}{12}(1,5,-6)$

Table 1: $\mathbb{Z}_N$ actions in $D = 4$.

are required. They fill the full space-time and six dimensional compact space. For N even, D5_k-branes, with worldvolume filling space-time and the kth complex plane, may be required. This is so whenever the orientifold group contains the element $\Omega R_i R_j$, for $k \neq i, j$. Here R_i (R_j) is an order two twist of the ith (jth) complex plane.

In what follows we consider cases with only one set of five branes. $\mathbb{Z}_N$ twists in table 1 were organized in such a way that, for even N, the order two element $R = \theta^{N/2}$ inverts the complex planes Y_1 and Y_2 and thus the corresponding orientifolds have D5_3-branes, filling space-time and compact dimension Y_3.

Open string states are denoted by $|\Psi, ab\rangle$, where Ψ refers to world-sheet degrees of freedom while the a, b Chan-Paton indices are associated to the open string endpoints lying on Dp-branes and Dq-branes respectively.

These Chan-Paton labels must be contracted with a hermitian matrix λ^{pq}_{ab}. The action of an element of the orientifold group on Chan-Paton factors is achieved by a unitary matrix $\gamma_{g,p}$ such that $g : \lambda^{pg} \to \gamma_{g,p} \lambda^{pq} \gamma^{-1}_{g,q}$. We denote by $\gamma_{k,p}$ the matrix associated to the $\mathbb{Z}_N$ orbifold twist θ^k acting on a Dp-brane.

Consistency under group transformations imposes restrictions on the representations γ_g. For instance, from $\Omega^2 = 1$ it follows that

$$\gamma_{\Omega,p} = \pm \gamma^T_{\Omega,p}\,. \tag{2.1}$$

Tadpole cancellation imposes further constraints on γ_g. Since we are planning to compare such restrictions with those coming from anomaly cancellation of gauge theories on D5 and D9-brane configurations, we will not impose the former in what follows, but just consider generic actions obeying the algebraic consistency conditions. Nevertheless, we will perform a definite choice of signs in (2.1), namely

$$\gamma_{\Omega,9} = \gamma^T_{\Omega,9}\,, \qquad \gamma_{\Omega,5} = -\gamma^T_{\Omega,5} \tag{2.2}$$

for Ω acting on 9 and on 5-branes. The first condition is the usual requirement of global consistency of the ten form potential in type I theory. Second equation is in agreement with the Gimon and Polchinski action, analyzed in [17]. These constraints lead to SO($2N_9$) and USp($2N_5$) groups in the 99 and 55 open string sectors respectively, where $2N_9$ ($2N_5$) is the number of D9(5)-branes (an even number is required by Ω action we will consider). When $N_9 = N_5 = 16$ such conditions ensure cancellation of untwisted tadpoles. Notice that consistency under the action of $(\Omega\theta^k)^2 = \theta^{2k}$ and (2.2) lead to

$$\gamma^*_{k,p} = \gamma_{\Omega,p}\,\gamma_{k,p}\,\gamma_{\Omega,p} \quad \text{for } p = 9, 5\,. \tag{2.3}$$

Thus, for a $\mathbb{Z}_N$ orbifold twist action, with $N = 2P$ ($N = 2P + 1$) for N even (odd), a generic matrix $\gamma_{\theta,p}$ can be written as

$$\gamma_{1,p} = \left(\tilde{\gamma}_{1,p}, \tilde{\gamma}^*_{1,p}\right)\,, \tag{2.4}$$

where $*$ denotes complex conjugation. $\tilde{\gamma}$ is a $N_p \times N_p$ diagonal matrix given by

$$\tilde{\gamma}_{1,p} = \mathrm{diag}\left(\ldots, \alpha^{NV_j} I_{n^p_j}, \ldots, \alpha^{NV_P} I_{n^p_P}\right) \tag{2.5}$$

with $\alpha = e^{2i\pi/N}$. $V_j = j/N$ with $j = 0, \ldots, P$ corresponds to an action "with vector structure" ($\gamma^N = 1$) while $V_j = (2j-1)/(2N)$ with $j = 1, \ldots, P$ describes an action "without vector structure" ($\gamma^N_{1,p} = -1$).[4] If we choose matrices $\gamma_{\Omega,9}$ and $\gamma_{\Omega,5}$

$$\gamma_{\Omega,9} = \begin{pmatrix} 0 & \mathrm{I}_{N_9} \\ \mathrm{I}_{N_9} & 0 \end{pmatrix}\;; \qquad \gamma_{\Omega,5} = \begin{pmatrix} 0 & -i\mathrm{I}_{N_5} \\ i\mathrm{I}_{N_5} & 0 \end{pmatrix} \tag{2.6}$$

then (2.2) and (2.3) are satisfied. In what follows we will be mainly concerned with actions "without vector structure" whenever D5-branes are present.[5] In those cases, the Chan-Paton matrices for the orbifold twist break the symplectic factors down to unitary groups.

For later convenience note that the trace of twist matrix above, or in general of its kth power $\gamma_{k,p}$, reads

$$\mathrm{Tr}\,\gamma_{k,p} = \sum_{j=0(1)}^{P} 2n^p_j \cos\left(2\pi V_j k\right)\,, \tag{2.7}$$

where the sum starts from 0(1) for the "with (without) vector structure" actions. In particular, for $k = 0$ we obtain $\mathrm{Tr}\, I_p = \sum 2n^p_j = 2N_p$, the number of D$p$-branes.

Moreover, in order to compare anomaly cancellation conditions, usually given in terms of the integers n^p_j, with tadpole equations, usually written in terms of above traces, it is also useful to have the former expressed in terms of the latter. This is easily achieved by performing a discrete Fourier transformation. For instance, for the "without vector structure" case, by multiplying both sides of eq. (2.7) by $\cos(2k\pi V_j)$

[4] Following the classification introduced in [20] for six-dimensional models.

[5] This is the case most widely studied for compact orientifolds although models with vector structure can also be constructed [11, 18].

and by summing over $k = 0, \ldots, N-1$ and using that $\operatorname{Tr} \gamma_{N-k,p} = -\operatorname{Tr} \gamma_{k,p}$, we obtain

$$n_j^p = \frac{1}{N} \left[\operatorname{Tr} \gamma_{0,p} + 2 \sum_{k=1}^{P} \operatorname{Tr} \gamma_{k,p} \cos\left(2V_j k\pi\right) \right] \tag{2.8}$$

for $j = 1, \ldots, P$. A similar expression is valid for the shift "with vector structure" (N odd) where we also have the $j = 0$ term

$$2n_0^p = \frac{1}{N} \left[\operatorname{Tr} \gamma_{0,p} + 2 \sum_{k=1}^{P} \operatorname{Tr} \gamma_{k,p} \right] . \tag{2.9}$$

The spectrum associated to the 9 and 5-brane orientifold configuration is easily obtained by working in a Cartan-Weyl basis (see [18]).

The gauge fields living on the worldvolume of a Dp-brane have associated Chan-Paton factors λ^p corresponding to the gauge group G_p with $G_9 = \mathrm{SO}(2N_9)$ and $G_5 = \mathrm{Sp}(2N_5)$. In Cartan-Weyl basis such generators are organized into charged generators $\lambda_a = E_a$, $a = 1, \ldots, \dim G_p - \operatorname{rank} G_p$, and Cartan algebra generators $\lambda_I = H_I$, $I = 1, \ldots, \operatorname{rank} G_p$, such that

$$[H_I, E_a] = \rho_I^a E_a \, , \tag{2.10}$$

where the $(\operatorname{rank} G_p)$-dimensional vector with components ρ_I^a is the root associated to the generator E_a.

The matrices $\gamma_{1,p}$ and its powers represent the action of the $\mathbb{Z}_N$ group on Chan-Paton factors, and they correspond to elements of a discrete subgroup of the Abelian group spanned by the Cartan generators. Hence, we can write

$$\gamma_{1,p} = e^{-2i\pi V^p \cdot H} \, . \tag{2.11}$$

Thus, this equation defines a $(\operatorname{rank} G_p)$-dimensional vector V^p with coordinates corresponding to the V_j's defined in (2.5) above. Cartan generators are represented as tensor products of σ_3 Pauli matrices. In such a description the massless states are easily found. Let us consider the case in which all 5-branes sit at the origin. In the pp sector the gauge group is obtained by selecting the root vectors satisfying

$$\rho^a \cdot V^p = 0 \bmod \mathbf{Z} \, , \tag{2.12}$$

while matter states correspond to charged generators with

$$\rho^a \cdot V^p = v_i \bmod \mathbf{Z} \, . \tag{2.13}$$

Recall that root vectors for orthogonal groups are of the form $\underline{(\pm 1, \pm 1, 0, \ldots, 0)}$ where underlining indicates that all possible permutations must be considered. In the symplectic case we have to include, in addition, the long roots $\underline{(\pm 2, 0, \ldots, 0)}$.

In the 95 sector the subset of roots of $G_9 \times G_5$ of the form

$$P_{(95)} = (W_{(9)}; W_{(5)}) = (\underline{\pm1, 0, \ldots, 0}; \underline{\pm1, 0, \ldots, 0}) \tag{2.14}$$

must be considered. Matter states are obtained from

$$P_{(95)} \cdot V^{(95)} = (s_j v_j + s_k v_k) \bmod \mathbf{Z} \tag{2.15}$$

with $s_j = s_k = \pm 1/2$, plus (minus) sign corresponding to particles (antiparticles) and $V^{95} = (V^9; V^5)$.

3. Tadpoles versus anomalies in $D = 4$, $N = 1$ orientifolds

3.1 Odd order orientifolds

In this subsection, we center on the study of $D = 4$ $N = 1$ $\mathbb{Z}_N$ orientifolds, with odd N. These models are consistent without the introduction of D5-branes, so only D9-branes are included. The only compact odd order orientifolds correspond to $\mathbb{Z}_3$, $\mathbb{Z}_7$, and $\mathbb{Z}_3 \times \mathbb{Z}_3$ with vector twists given in table 1. However, let us momentarily be more general and consider also $\mathbb{Z}_N$ orbifold actions which are not necessarily crystallographic. Thus, we focus on twist generators θ with eigenvalues $(1/N)(t_1, t_2, t_3)$ with $\sum_{a=1}^{3} t_a = 0 \bmod N$, and $N = 2P + 1$.

The strategy to study the relation between the anomaly and tadpole cancellation conditions will be as follows. First we compute the gauge group and massless matter for a general such model. In this step only algebraic consistency conditions (group law) are imposed on the Chan-Paton matrices. Next we find generic conditions for cancellation of gauge anomalies.[6] Since for SU(n) groups only fundamental $\mathbf{n}$ and/or antisymmetric $\mathbf{a}_n$ representations (or their conjugates) appear, such conditions will manifest as restriction on the group ranks in order to ensure that only anomaly-free combinations are allowed. Finally, these restrictions are compared with tadpole cancellation equations obtained from type IIB orientifolds.

In agreement with eq. (2.5), the generic action of the orbifold twist on 9-branes can be encoded in the matrix

$$\gamma_{1,9} = (\tilde{\gamma}_{1,9}, \tilde{\gamma}^*_{1,9}) \tag{3.1}$$

with $\tilde{\gamma}$ given by

$$\tilde{\gamma} = \mathrm{diag}\left(I_{n_0^9}, \alpha I_{n_1^9}, \ldots, \alpha^j I_{n_j^9}, \ldots, \alpha^P I_{n_P^9}\right) \tag{3.2}$$

[6]Notice that when one of the gauge factors is absent, the conditions of anomaly cancellation may be less restrictive than those we consider [2]. However, this case is rather particular, and we find it is more insightful to consider 'generic' cancellation of anomalies, as we do in the present paper.

and $\alpha = e^{2i\pi/N}$. The associated shift is

$$V^9 = \frac{1}{N}\Big(0,\dots,0,1\dots 1,\dots,j,\dots,j,\dots,P,\dots,P\Big) \tag{3.3}$$

from where we can easily read the gauge group to be

$$\mathrm{SO}(u_0) \times \prod_{j=1}^{P} \mathrm{U}(u_j)\,, \tag{3.4}$$

where we have defined $u_0 = 2n_0^9$ and $u_j = n_j^9$. For the sake of clarity let us first consider orbifold twists of the form $(1/N)(1,1,-2)$.

The corresponding massless spectrum is

$$2\left[\overline{\mathbf{a}}_P + (\mathbf{u}_0,\mathbf{u}_1)_{(1)} + \sum_{j=1}^{P-1}\big(\overline{\mathbf{u}}_j,\mathbf{u}_{j+1}\big)_{(-1,1)}\right] + \overline{\mathbf{a}}_1 + (\mathbf{u}_0,\overline{\mathbf{u}}_2)_{(-1)} + \\ + \sum_{j=1}^{P-2}\big(\mathbf{u}_j,\overline{\mathbf{u}}_{j+2}\big)_{(-1,1)} + (\mathbf{u}_{P-1},\mathbf{u}_P)_{(1,1)}\,, \tag{3.5}$$

where, inside brackets we have indicated the charge with respect to the U(1) factor in $\mathrm{U}(u_j)$. The absence of $\mathrm{SU}(u_j)$ gauge anomalies requires

$$\begin{aligned} &\mathrm{SU}(u_1):\ 2u_0 + u_3 - 2u_2 - u_1 + 4 = 0\,,\\ &\mathrm{SU}(u_j):\ 2u_{j-1} + u_{j+2} - 2u_{j+1} - u_{j-2} = 0\,,\\ &\mathrm{SU}(u_P):\ 3u_{P-1} - u_{P-2} - 2u_P + 8 = 0\,, \end{aligned} \tag{3.6}$$

where $j \neq 1, P$. Actually, by performing the identifications $u_j = u_{-j}$ and $u_{N+k} = u_k$ these conditions can be recast into the unique expression

$$\mathrm{SU}(u_j):\ 2u_{j-1} + u_{j+2} - 2u_{j+1} - u_{j-2} = -4\delta_{j,1} - 8\delta_{j,P} \tag{3.7}$$

for $j = 1, P$.

Moreover, it is not difficult to generalize these equations to the case of general odd order orbifolds, generated by a twist with eigenvalues $(1/N)(t_1,t_2,t_3)$, with $\sum t_a = 0 \bmod N$. We obtain

$$\mathrm{SU}(u_j):\sum_{a=1}^{3}(u_{j-t_a} - u_{j+t_a}) = 4\sum_{a=1}^{3}(\delta_{2j,t_a} - \delta_{2j,-t_a}) \tag{3.8}$$

for $j = 1,\dots,P$, and the arguments of the Kronecker deltas are defined $\bmod N$.

Such anomaly cancellation conditions can be reexpressed in terms of traces of the matrix γ by using eqs. (2.8) and (2.9) that, with the above definition of u_0 now read

$$u_j = \frac{1}{N}\left[\mathrm{Tr}\,\gamma_{0,9} + 2\sum_{k=1}^{P}\mathrm{Tr}\,\gamma_{k,9}\cos\left(\frac{2kj\pi}{N}\right)\right] \tag{3.9}$$

for $j = 0,\dots,P$.

Hence, by replacing in (3.8) we obtain

$$\frac{1}{N}\sum_{k=1}^{P}\sin\left(\frac{2\pi kj}{N}\right)\left[\sum_{a=1}^{3}\sin\left(\frac{2\pi kt_a}{N}\right)\right]\mathrm{Tr}\,\gamma_{k,9}=\sum_{a=1}^{3}(\delta_{2j,t_a}-\delta_{2j,-t_a}). \tag{3.10}$$

By Fourier transforming the delta functions and using that

$$\sum_{a=1}^{3}\sin\left(\frac{2\pi kt_a}{N}\right)=-4\prod_{a=1}^{3}\sin\left(\frac{\pi kt_a}{N}\right) \tag{3.11}$$

we obtain the general conditions for the absence of gauge anomalies

$$\prod_{a=1}^{3}\sin\left(\frac{\pi kt_a}{N}\right)\left[2\,\mathrm{Tr}\,\gamma_{2k,9}\prod_{a=1}^{3}\cos\left(\frac{\pi kt_a}{M}\right)-1\right]=0\,. \tag{3.12}$$

Notice that even if $\mathrm{Tr}\,\gamma_{0,9}$ appears in (3.9), it is not present in these conditions, there is no dependence on the total number of 9-branes. This was expected, given the relation of these gauge theories to systems of D3-branes at non-compact singularities, to be discussed below.

For the compact $\mathbb{Z}_3$ and $\mathbb{Z}_7$ cases these are the well known twisted tadpole cancellation conditions $\mathrm{Tr}\,\gamma_{1,9}=-4$ and $\mathrm{Tr}\,\gamma_2=4$ respectively [21, 22, 18] . Such conditions on the traces or, equivalently, eqs. (3.8), completely fix the values of u_j's, and thus lead to the unique solutions found in the literature, when the number of 9-branes is 32.

In order to interpret several results in this paper, it will be useful to relate our models to systems of D3 branes at orientifold singularities [11, 12, 13, 14]. The basic observation is that the gauge theory we have described in the case at hand can be realized by placing D3 branes at a non-compact $\mathbb{Z}_N$ orientifold singularity [14]. This can be understood by T-dualizing along the three compact directions, which transforms the D9-branes into D3-branes sitting at one of the fixed points, and then taking a decompactification limit. Thus, the conditions (3.12) ensure the cancellation of anomalies on the D3 brane worldvolume. This explains why $\mathrm{Tr}\,\gamma_0$ (which in this case is the total number of D3 branes) is unconstrained: There is an infinite family of non-anomalous theories, parametrized by the number of D3 branes at the singularity. Actually, in the non-compact case, the conditions above are exactly equivalent to the tadpole cancellation conditions, in analogy with the result in [2]. We have just seen that in the particular case of $\mathbb{Z}_N$ (N odd) orientifolds, compact models also have this property.

The same conclusion, however, does not follow for other types of orientifold, as we show below. To this end, let us consider for instance orientifolds $\mathbb{Z}_{N_1}\times\mathbb{Z}_{N_2}$, with N_1, N_2 odd. As in the above models, these theories are consistent without the addition of D5-branes. The analysis of the relation between anomaly cancellation and tadpole cancellation conditions can be studied following the strategy used above, so we will be more sketchy, leaving the details for specific examples. The first step is to compute the spectrum on the D9-brane sector for a general $\mathbb{Z}_{N_1}\times\mathbb{Z}_{N_2}$ model, and compute its non-Abelian anomalies in terms of the ranks of the group factors. Then we perform a discrete Fourier transform to rewrite them in terms of Chan-Paton matrices associated to the twists $(1/N)(t_1,t_2,t_3)$ in the orientifold group. The resulting anomaly cancellation conditions have exactly the form (3.12) (It must be understood that the matrix γ in (3.12) will correspond to a product of powers of γ_θ and γ_ω associated to the particular twist $(1/N)(t_1,t_2,t_3)$, where θ and ω are the $\mathbb{Z}_{N_1}$, $\mathbb{Z}_{N_2}$ generators, respectively).

The fact that we obtain the same expression for $\mathbb{Z}_N$ and $\mathbb{Z}_{N_1}\times\mathbb{Z}_{N_2}$ orientifolds (with odd N, N_1, N_2) is related to the fact that the twists in a $\mathbb{Z}_{N_1}\times\mathbb{Z}_{N_2}$ orientifold and in $\mathbb{Z}_N$ orientifolds have the same structure (namely, no order two twist is contained in the group).

In particular the expression (3.12) holds for the compact $\mathbb{Z}_3\times\mathbb{Z}_3$ orientifold with θ and ω described by the eigenvalues $(1/3)(1,-1,0)$ and $(1/3)(0,1,-1)$. It is important to observe that the first coefficient in (3.12) vanishes when one direction is not affected by the twist. Therefore, for the case under consideration, only one constraint corresponding to the twist $\theta\omega^2$ with eigenvalues $(1/3)(1,1,-2)$, thus affecting all three complex directions, is found. It reads

$$\mathrm{Tr}\,\gamma_\theta\gamma_{\omega^2} = -4\,. \tag{3.13}$$

Hence, we find that anomaly cancellation is much less restrictive, in this case, than tadpole cancellation, which also requires [22, 23] the set of equations

$$\mathrm{Tr}\,\gamma_\theta = \mathrm{Tr}\,\gamma_\omega = \mathrm{Tr}\,\gamma_{\theta\omega} = 8 \tag{3.14}$$

associated to the other twists, to be satisfied. Imposing these additional conditions, the spectrum of the model is completely fixed. Thus, even if a whole set of $\mathbb{Z}_3\times\mathbb{Z}_3$ anomaly free models satisfying (3.13) can be constructed, there is, however, a unique solution satisfying all tadpole cancellation conditions.

Let us consider this case in more detail. Twists θ and ω are represented by the matrices

$$\begin{aligned}\tilde\gamma_\theta &= \mathrm{diag}(\mathrm{I}_{w_0},\mathrm{I}_{u_1},\alpha\mathrm{I}_{u_2},\alpha^2\mathrm{I}_{u_3},\alpha\mathrm{I}_{u_4})\,,\\ \tilde\gamma_\omega &= \mathrm{diag}(\mathrm{I}_{w_0},\alpha\mathrm{I}_{u_1},\mathrm{I}_{u_2},\mathrm{I}_{u_3},\alpha^2\mathrm{I}_{u_4})\end{aligned} \tag{3.15}$$

with $\alpha = e^{2i\pi/3}$.

The gauge group is $\mathrm{SO}(2w_0) \times \mathrm{U}(u_1) \cdots \mathrm{U}(u_4)$ and the massless spectrum can be easily computed to be

$$\begin{aligned}
&\overline{\mathbf{a}}_2 + (\overline{\mathbf{u}}_1, \mathbf{u}_3) + (\mathbf{u}_1, \mathbf{u}_4) + (\overline{\mathbf{u}}_3, \overline{\mathbf{u}}_4) + (\mathbf{2w}_0, \mathbf{u}_2), \\
&\mathbf{a}_4 + (\overline{\mathbf{u}}_1, \overline{\mathbf{u}}_3) + (\mathbf{u}_1, \overline{\mathbf{u}}_2) + (\mathbf{u}_2, \mathbf{u}_3) + (\mathbf{2w}_0, \overline{\mathbf{u}}_4), \\
&\mathbf{a}_1 + (\overline{\mathbf{u}}_2, \mathbf{u}_4) + (\mathbf{u}_3, \overline{\mathbf{u}}_4) + (\mathbf{u}_2, \overline{\mathbf{u}}_3) + (\mathbf{2w}_0, \overline{\mathbf{u}}_1).
\end{aligned} \tag{3.16}$$

If we only impose the anomaly cancellation condition

$$\mathrm{Tr}\, \gamma_\theta \gamma_{\omega^2} = 2(w_0 + u_3) - u_1 - u_2 - u_4 = -4 \tag{3.17}$$

many anomaly-free spectra can be obtained. Indeed, as discussed above, these models can be related to systems of D3-branes at $\mathbb{Z}_3 \times \mathbb{Z}_3$ singularities. For these *non-compact* constructions, all these models are consistent. In the non-compact case the reason why the tadpole conditions (3.14) need not be imposed is clear [2]. Those conditions correspond to twists which leave one complex plane unrotated. Thus the RR flux can escape to infinity through those planes and one does not need to impose the corresponding tadpole cancellation. In the compact case this is different: Even though those planes are unrotated they are compact and the RR charge is trapped. Thus the twisted RR charge corresponding to those twists has to be canceled and the conditions (3.14) have to be imposed. However, although these conditions are needed for consistency, they are totally disconnected from the issue of gauge anomaly cancellation.

3.2 Even order orientifolds

For orientifolds including even order twists, there will be in general both D9-branes and D5-branes present. In this section, we consider the construction of $D = 4$, $N = 1$ models obtained from orientifold configurations of D9 and D5-branes.

As in the previous section, we first consider a general (not necessarily crystallographic) orbifold twist. In a first step we compute the gauge group and massless spectrum of such models for an arbitrary number of $2N_9$ ($2N_5$) D9(D5)-branes. Generic conditions for cancellation of gauge anomalies are then found. Using the discrete Fourier transform, these restrictions are then compared with tadpole cancellation equations obtained from type IIB orientifolds. Finally, explicit solutions to these equations are discussed. In some cases new solutions, other than those reported in the literature are found. For simplicity we concentrate on models with only one set of D5-branes, all of them sitting at the fixed point at the origin. Other situations are considered through specific examples.

Let us consider arbitrary $\mathbb{Z}_N (N = 2P)$ twists with eigenvalues given by $(1/N)(t_1, t_2, t_3)$ with $t_1 + t_2 + t_3 = 0$ and t_3 an even integer (thus t_1, t_2 are odd).

We concentrate on models without vector structure. Thus, from eq. (2.5) we have

$$\tilde{\gamma} = \mathrm{diag}\Big(\alpha I_{n_1}, \ldots, \alpha^{(2j-1)} I_{n_j}, \ldots, \alpha^{(2P-1)} I_{n_P}\Big) \tag{3.18}$$

with $\alpha = e^{i\pi/N}$ and where we have dropped the upperindex 5 ($n_j^5 = n_j$).

A similar structure is used in the 9-brane sector just by replacing the integers n_j's by u_j's. These matrices correspond to the shifts

$$V^p = \frac{1}{2N}\Big(1, \ldots 1, \ldots, 2j-1, \ldots, 2j-1, \ldots, 2P-1, \ldots, 2P-1\Big) \tag{3.19}$$

with $n_j(u_j)$ entries $(2j-1)$ for $p = 5(9)$, with $j = 1, \ldots, P$.

The resulting gauge group is

$$\prod_{j=1}^{P} \mathrm{U}(n_j) \times \prod_{j=1}^{P} \mathrm{U}(u_j). \tag{3.20}$$

The massless states in 55 sector are

$$\begin{aligned} \sum_{a=1}^{3} {\sum_{j=1}^{P}}' \Big[& (\overline{\mathbf{n}}_j, \mathbf{n}_{j+t_a})_{(-1,1)} + \big(\overline{\mathbf{n}}_j, \overline{\mathbf{n}}_{[-j-t_a+1]}\big)_{(-1,-1)} + (\mathbf{n}_j, \overline{\mathbf{n}}_{j-t_a})_{(1,-1)} + \\ & + \big(\mathbf{n}_j, \mathbf{n}_{[-j+t_a+1]}\big)_{(1,1)}\Big] + \sum_{j=1}^{P}\Big[\mathbf{a}_{j_{(2)}}\big(\delta_{j,(t_1+1)/2} + \delta_{j,(N+t_1+1)/2}\big) + \\ & + \overline{\mathbf{a}}_{j_{(-2)}}\big(\delta_{j,(-t_1+1)/2} + \delta_{j,(N-t_1+1)/2}\big) + (t_1 \to t_2)\Big] \end{aligned} \tag{3.21}$$

while the 59 sector spectrum is

$$\begin{aligned} \sum_{j=1}^{P} & (\overline{\mathbf{n}}_j, \mathbf{u}_{j+t})_{(-1,1)} + \big(\overline{\mathbf{n}}_j, \overline{\mathbf{u}}_{[-j-t+1]}\big)_{(-1,-1)} + (\mathbf{n}_j, \overline{\mathbf{u}}_{j-t})_{(1,-1)} + \\ & + \big(\mathbf{n}_j, \mathbf{u}_{[-j+t+1]}\big)_{(1,1)} + (\mathbf{n} \leftrightarrow \mathbf{u}) \end{aligned} \tag{3.22}$$

where the indices j, j' are defined mod N. Notice that sums over bifundamentals with indices (j, j') must be performed such that $1 \le j \le j' \le (N/2)$. Also, the primed sum on 55 sector means that $j = j'$ is not included. In fact, in this case, antisymmetric representations do appear for t_a odd, i.e. $a = 1, 2$ for our convention above. In 59 sector we have defined $t = (t_1 + t_2)/2$, see (2.15). The spectrum in 99-sector can be obtained from eq. (3.21) by replacing n's by u's.

Again, it is straightforward to write down a general condition for the cancellation of non-Abelian anomalies. It reads

$$\begin{aligned} \mathrm{SU}(n_j) : \sum_{a=1}^{3} & \big[(n_{j-t_a} - n_{j+t_a}) + u_{j-t} - u_{j+t}\big] = \\ & = 4\big[\delta_{j,(t_1+1)/2} + \delta_{j,(N+t_1+1)/2} - \delta_{j,(-t_1+1)/2} - \delta_{j,(N-t_1+1)/2} + (t_1 \to t_2)\big] \end{aligned} \tag{3.23}$$

with $j = 1, \ldots, P$ and where the identifications $n_j = n_{-j+1} = n_{j+N}$ are understood. As in the odd twist case, such equivalences are automatically implemented when n_j's (u_j's) are expressed in terms of traces through eq. (2.8). In fact, by using such equation above we find that, in order to ensure the absence of gauge anomalies for the 55 group sector we must have

$$\frac{1}{N}\sum_{k=1}^{P-1} \sin(2\pi k V_j)\left[\sin\left(\frac{t_3\pi k}{N}\right)A_k\right] = \\ = -\delta_{j,(t_1+1)/2} - \delta_{j,(N+t_1+1)/2} + \delta_{j,(-t_1+1)/2} + \delta_{j,(N-t_1+1)/2} + (t_1 \to t_2) \qquad (3.24)$$

with

$$A_k = 4\sin\left(\frac{t_1\pi k}{N}\right)\sin\left(\frac{t_2\pi k}{N}\right)\mathrm{Tr}\,\gamma_{k,5,0} + \mathrm{Tr}\,\gamma_{k,9}\,, \qquad (3.25)$$

where we have emphasized, by adding a 0 subscript, that D5-branes are located at the origin. Again δ functions appear from the contribution of antisymmetric representations. Notice that the sum is actually up to $P-1$ (and not up to P). This is due to the fact that, from our definition (3.18) of the twist matrix, in the case without vector structure,

$$\mathrm{Tr}\,\gamma_{P,p} = 0\,. \qquad (3.26)$$

This equation for the order two twist matrix, which in this case is automatically satisfied by construction, appears in [18, eq. (2.39)] as necessary condition for the cancellation of tadpoles.

Again, it will be useful to keep in mind that these gauge theories can be realized on the worldvolume of D3-branes at $\mathbb{Z}_N$ orientifold singularities. In this case, D7-branes are also present in the configuration. In this non-compact context, it is possible to show that the anomaly cancellation conditions (3.24) are exactly equivalent to the tadpole cancellation conditions. In the following we show that this property is in general no longer true in the compact models.

The constraints that we have just obtained may be read as a set of P equations (for each value of j) with $P-1$ unknowns $\sin((t_3\pi k)/N)A_k$. Thus, the system is, in principle, overdetermined and it could have no solutions at all, unless not all equations are really independent. In fact, we will see that some models are not consistent.

Recall that a similar set of equations, obtained from above simply by exchanging $\gamma_{k,9}$ and $\gamma_{k,5}$ would be required in order to avoid gauge anomalies in the 99 sector groups. This $5 \leftrightarrow 9$ symmetry is due to the fact that all fivebranes have been put at the origin, and it is a manifestation of T (self) duality. Nevertheless we will see that despite the symmetry of anomaly equations, in some cases they allow for solutions which are not symmetric under the exchange $5 \leftrightarrow 9$. The additional tadpole cancellation equation leads to a fully T duality invariant spectrum.

The above constraints are valid for an arbitrary, not necessarily chrystalographic, even twist action on 5 and 9-branes. In what follows we analyze the compact cases shown in table 1. The global constraint $\mathrm{Tr}\,\gamma_{0,9} = \mathrm{Tr}\,\gamma_{0,5} = 32$ must be imposed in such cases.

3.2.1 Non-consistent models

For orbifold groups containing a twist with eigenvalues $(1/4)(1,1,-2)$, namely the orbifold actions $\mathbb{Z}_4$, $\mathbb{Z}_8$, $\mathbb{Z}_8{}'$, and $\mathbb{Z}_{12}{}'$ the above equations have no solutions. In fact, these orientifolds were found in [18] to be ill-defined. Difficulties stem from the presence of a Klein-bottle tadpole proportional to the volume V_3 of the third compact dimension.

Interestingly enough, such inconsistencies are recovered here from the point of view of anomaly cancellation. This can be easily seen in the $\mathbb{Z}_4$ case. In fact, (3.24) leads to the incompatible equations $(\sqrt{2}/2)A_1 = 8$ for $j = 1$ and $(\sqrt{2}/2)A_1 = -8$ for $j = 2$ where $A_1 = 2\,\mathrm{Tr}\,\gamma_{1,5,0} + \mathrm{Tr}\,\gamma_{1,9}$. The same situation repeats in the other cases.

We should emphasize that the above comments for inconsistency refer only to Chan-Paton twists without vector structure. It is in principle possible to find consistent $D = 4$, $N = 1$ orientifolds corresponding to these twists but with CP twists with vector structure. Indeed we have found consistent (non-chiral) examples based on $\mathbb{Z}_4$ with vector structure.

3.2.2 Consistent models

For all other twists in table 1 it is possible to write down a general, consistent, solution to eq. (3.24). Namely

$$\sin\left(\frac{t_3\pi k}{N}\right) A_k = -4(1+(-1)^k)\left[\sin\left(\frac{t_1\pi k}{N}\right) + \sin\left(\frac{t_2\pi k}{N}\right)\right] \tag{3.27}$$

with $k = 1, \ldots, P$. Thus, for each twist θ^k of order k we obtain a condition which relates $\mathrm{Tr}\,\gamma_{k,5}$ and $\mathrm{Tr}\,\gamma_{k,9}$ as they show up in tadpole cancellation equations. Whenever k is such that θ^k twist leaves the direction parallel to the D5-brane untouched, then $\sin((t_3\pi k)/N) = 0$ (thus also $\sin((t_1\pi k)/N) + \sin((t_2\pi k)/N) = 0$) and no constraint is present for the corresponding $\mathrm{Tr}\,\gamma_{k,9}$ and $\mathrm{Tr}\,\gamma_{k,5}$.

On the other hand, since we must have a similar solution for anomaly cancellation in 99 sector groups but with $5 \to 9$ in (3.25), we will have

$$\left[4\sin\left(\frac{t_1\pi k}{N}\right)\sin\left(\frac{t_2\pi k}{N}\right) - 1\right](\mathrm{Tr}\,\gamma_{k,5,0} - \mathrm{Tr}\,\gamma_{k,9}) = 0 \tag{3.28}$$

for all other twists with $t_3\pi k \neq 0 \bmod N$. Notice that $4\sin((t_1\pi k)/N)\sin((t_2\pi k)/N)$ is, by the Lefschetz fixed point theorem, nothing but the number of fixed points of twist θ^k in the complex directions (Y_1, Y_2). Thus, this equation implies that whenever points other than the origin are kept fixed then $\mathrm{Tr}\,\gamma_{k,5,0} = \mathrm{Tr}\,\gamma_{k,9}$. Also notice that only even values of k could lead to non-zero values for A_k.

Let us consider the different cases in more detail.

The case of $\mathbb{Z}_6$: from eq. (3.27) we find $A_1 = 0$ and $A_2 = 16$, namely

$$\mathrm{Tr}\,\gamma_{1,9} + \mathrm{Tr}\,\gamma_{1,5,0} = 0\,, \qquad \mathrm{Tr}\,\gamma_{2,9} + 3\,\mathrm{Tr}\,\gamma_{2,5,0} = 16\,. \tag{3.29}$$

These are tadpole cancellation conditions found in [22, 18]. However, in ref. [18] it is found that tadpole cancellation at other fixed points, other than the origin, imposes an extra requirement. Namely

$$\mathrm{Tr}\,\gamma_{2,9} + 3\,\mathrm{Tr}\,\gamma_{2,5,J} = 4 \tag{3.30}$$

with $J = 1, \ldots, 8$ denoting the fixed points of θ^2 in the (Y_1, Y_2) planes, other than the origin, where 5-branes may sit. Since here all branes are at the origin then $\mathrm{Tr}\,\gamma_{2,5,J} = 0$. We are led to

$$\mathrm{Tr}\,\gamma_{2,9} = 4\,. \tag{3.31}$$

This is precisely the extra condition (3.28) that results when anomaly cancellation for the 99 sector groups is required.

Thus, we learn that for the $\mathbb{Z}_6$ orientifold, generic absence of anomalies and of tadpole divergences are equivalent. Let us stress that to arrive at this conclusion, analysis of tadpole cancellation at all fixed points (and not only at the origin) is needed. In fact, it is eq. (3.31) which makes tadpole equations to look symmetric under the exchange of 9 and 5-branes (all at the origin) and thus ensures cancellation of 99 sector anomalies. It appears to us that this point is not sufficiently clear in the literature. Sometimes tadpole equations asymmetric between D9 and D5-branes are written down for four- or six-dimensional orientifolds. These are misleading since, as they stand, they would allow for 5-9 asymmetric solutions which could be anomalous. Taking into account tadpole cancellations in fixed points away from the origin does in fact impose D9-D5 symmetric solutions. This is just the right behaviour, given the fact that putting all 5-branes at the origin is a selfdual configuration under T-duality.

Let us now look for solutions of the above equations. An interesting feature is that the first row in (3.29) indicates the possibility of a local cancellation among 9-branes and 5-branes RR charges that may not vanish independently. This possibility has not been noticed in the literature, and we will exploit it here to construct new solutions of the tadpole equations for all 5-branes at origin. The new possibilities, as we saw in eq. (3.28), have to do with the fact that only the origin is fixed under $\mathbb{Z}_6$ twist.

In order to study the solutions it is easier to rewrite the traces in terms of n_j's and u_j's. Hence, by using (2.7) we have

$$\frac{1}{\sqrt{3}}\,\mathrm{Tr}\,\gamma_{1,5} = n_1 - n_3\,, \qquad \mathrm{Tr}\,\gamma_{2,5} = n_1 - 2n_2 + n_3 \tag{3.32}$$

and similar equations for $\mathrm{Tr}\,\gamma_{k,9}$ in terms of u_j's. Replacing (3.29), (3.30), (3.31) and using that there are 32 D5- and D9-branes we obtain

$$u_1 = 12 - n_1\,, \qquad u_2 = n_2 = 4\,, \qquad u_3 = n_1\,. \tag{3.33}$$

Therefore, from (3.20), the gauge groups, depending on the free integer parameter $n_1 \leq 12$, are

$$\left[\mathrm{U}(n_1)\times \mathrm{U}(4)\times \mathrm{U}(12-n_1)\right]_{55} \times \left[\mathrm{U}(12-n_1)\times \mathrm{U}(4)\times \mathrm{U}(n_1)\right]_{99}\,. \tag{3.34}$$

The matter content may easily be obtained from (3.21) and (3.22). Notice that the solution given in [22, 18] has $n_1 = n_3 = 6$ implying that $\mathrm{Tr}\,\gamma_{1,5} = \mathrm{Tr}\,\gamma_{1,9} = 0$. Therefore it corresponds to the case in which 5 and 9 brane contributions cancel independently.

The case of $\mathbb{Z}_6{}'$: absence of anomalies in the 55 sector leads to

$$\mathrm{Tr}\,\gamma_{1,9} - 2\,\mathrm{Tr}\,\gamma_{1,5,0} = 0\,, \qquad \mathrm{Tr}\,\gamma_{2,9} = -8\,, \tag{3.35}$$

which again coincide with equations for tadpole cancellation at the origin. When the anomaly cancellation conditions from the 99 sector are included the full set of constraints

$$\mathrm{Tr}\,\gamma_{1,9} = \mathrm{Tr}\,\gamma_{1,5} = 0\,, \qquad \mathrm{Tr}\,\gamma_{2,9} = \mathrm{Tr}\,\gamma_{2,5} = -8 \tag{3.36}$$

is obtained. It is completely equivalent to cancellation of all twisted tadpoles, including those at the other three fixed points in the transverse directions.

One aspect of this model may appear puzzling. Since the order three twist leaves one complex plane unrotated, one would expect, as it happened in the $\mathbb{Z}_3 \times \mathbb{Z}_3$ case, that the tadpole constraint for $\gamma_{2,9}$ is not required for anomaly cancellation. It turns out that the presence of D5-branes makes things different. Indeed, cancellation of gauge anomalies for 99 sector groups requires eq. (3.27) but with 9 and 5 indices exchanged in (3.25). Explicitly,

$$A_k = 4\sin\left(\frac{\pi k}{6}\right)\sin\left(\frac{-3\pi k}{6}\right)\mathrm{Tr}\,\gamma_{k,9} + \mathrm{Tr}\,\gamma_{k,5} \tag{3.37}$$

and thus, in effect, no constraint on $\mathrm{Tr}\,\gamma_{k,9}$ is found for $k=2$. Nevertheless $\mathrm{Tr}\,\gamma_{2,5} = -8$ is still required. The condition on $\mathrm{Tr}\,\gamma_{2,9}$ is really obtained from the 55 sector consistency. A further discussion on this point is presented in section 5.

Unlike $\mathbb{Z}_6$, in this case charges among 9-branes and 5-branes must cancel independently. This is due, essentially, to the presence of extra fixed points. As in the odd orientifold cases, since now the number of conditions on the traces and the number of unknowns are the same, the solution presented in [23, 18] (for all 5-branes at the origin) is unique.

It is instructive to analyze, from the point of view of anomaly cancellation, the situation when part of the 32 D5-branes live at other fixed points. As an example consider the case when five branes are distributed in groups of N_L^5 branes among the four fixed points $L = 0, \ldots, 3$ of the θ twist in the first two planes. The structure of the 55_L gauge group and matter content at point L is exactly the same as that for the origin considered above. The spectrum at each point is obtained from (3.21) and (3.22) just by replacing $n \to n^L$, with the condition $\sum_{L=0}^{3}(n_1^L + n_2^L + n_3^L) = 16$. Thus the anomaly constraints eq. (3.35) must now be imposed at each point. Namely

$$\operatorname{Tr}\gamma_{1,9} - 2\operatorname{Tr}\gamma_{1,5,L} = 0\,, \qquad \operatorname{Tr}\gamma_{2,9} = -8 \tag{3.38}$$

for $L = 0, \ldots, 3$.

However, constraints involving several fixed points are obtained when the 99 group is considered. The 59 sector (3.22) reads

$$\sum_{L=0}^{3}\Big[(\overline{\mathbf{u}}_1, \overline{\mathbf{n}}_1^L) + (\mathbf{u}_1, \overline{\mathbf{n}}_2^L) + (\mathbf{u}_2, \overline{\mathbf{n}}_3^L) + (\mathbf{u}_3, \mathbf{n}_3^L) + (\overline{\mathbf{u}}_3, \mathbf{n}_2^L) + (\overline{\mathbf{u}}_2, \mathbf{n}_1^L)\Big] \tag{3.39}$$

and so, a sum of contributions from all fixed points supporting D5-branes appears. Thus anomaly constraints will read

$$\sum_{L=0}^{3}\operatorname{Tr}\gamma_{1,5,L} - 2\operatorname{Tr}\gamma_{1,9} = 0\,, \qquad \sum_{L=0}^{3}\operatorname{Tr}\gamma_{2,5,L} = -8\,. \tag{3.40}$$

Equations (3.38) and (3.40) reproduce the tadpole cancellation equations for this general case. Since D5-branes are now distributed among different fixed points we expect to be able to achieve charge cancellation in different ways rather than in the unique form found above. For instance, first row in equations above tells us that now it could still be possible to achieve charge cancellation among 9 and 5-branes even if they did not vanish independently. By writin g the traces in terms of group ranks (2.7) and recalling that $u_1 + u_2 + u_3 = 16$, $\sum_{L=0}^{3} n_1^L + n_2^L + n_3^L = 16$ we find the following consistency constraints

$$n_1^L - n_3^L = u_1 - 4 = 4 - u_3\,, \qquad u_2 = \sum_{L=0}^{3} n_2^L = 8\,, \qquad \sum_{L=0}^{3} n_1^L + n_3^L = 8\,. \tag{3.41}$$

The case of $\mathbb{Z}_{12}$: in this example tadpole cancellation appears to be stronger than anomaly cancellation. Indeed, anomaly cancellation in the 55 sector gives

$$\begin{aligned} A_k &= \mathrm{Tr}\,\gamma_{k,9} - \mathrm{Tr}\,\gamma_{k,5,0} = 0; \quad k = 1,2,5\,, \\ A_4 &= \mathrm{Tr}\,\gamma_{4,9} + 3\,\mathrm{Tr}\,\gamma_{4,5,0} = 16\,, \end{aligned} \tag{3.42}$$

which must be supplemented with the extra, independent, constraint from the equivalent equations in the 99 sector,

$$\mathrm{Tr}\,\gamma_{4,9} = \mathrm{Tr}\,\gamma_{4,5,0} = 4\,. \tag{3.43}$$

Notice, however, that there is no constraint for $A_3 = \mathrm{Tr}\,\gamma_{3,9} + 2\,\mathrm{Tr}\,\gamma_{3,5,0}$, due to the appearance of the vanishing factor $\sin((4\pi k)/12)$ (for $k = 3$) associated to the fourth order twist $1/(4(1,-1,0))$ which leaves the third plane invariant.

Thus, we expect the model will have additional tadpole conditions, not needed to ensure anomaly cancellation. This additional constraint is the tadpole condition (2.45) of [18]

$$\mathrm{Tr}\,\gamma_{3,9} + 2\,\mathrm{Tr}\,\gamma_{3,5,L} = 0\,, \tag{3.44}$$

where L refers to the fixed points of θ^3 in the first and second complex coordinates. In our specific case with all D5 branes at the origin, the condition reads

$$\mathrm{Tr}\,\gamma_{3,9} = \mathrm{Tr}\,\gamma_{3,5,0} = 0\,. \tag{3.45}$$

These equations constrain the model beyond mere anomaly cancellation. This resembles much what happened with $\mathbb{Z}_3 \times \mathbb{Z}_3$.

Again, due to the fact that only the origin is fixed under $\mathbb{Z}_{12}$ there are more solutions to the tadpole equations than those in the literature.

Let us be more explicit. The generic gauge group is $\prod_{i=1}^{6} \mathrm{U}(n_i)$ while the 55+59 massless spectra is given by (3.21), (3.22)

$$\mathbf{a}_1 + \overline{\mathbf{a}}_6 + (\overline{\mathbf{n}}_1, \mathbf{n}_2) + (\overline{\mathbf{n}}_2, \mathbf{n}_3) + (\overline{\mathbf{n}}_3, \mathbf{n}_4) + (\overline{\mathbf{n}}_4, \mathbf{n}_5) + (\overline{\mathbf{n}}_5, \mathbf{n}_6), \tag{3.46}$$

$$\begin{aligned} &\overline{\mathbf{a}}_3 + \mathbf{a}_4 + (\overline{\mathbf{n}}_1, \overline{\mathbf{n}}_5) + (\overline{\mathbf{n}}_2, \overline{\mathbf{n}}_4) + (\mathbf{n}_2, \mathbf{n}_6) + (\mathbf{n}_3, \mathbf{n}_5) + (\mathbf{n}_1; \overline{\mathbf{n}}_6) + \\ &\quad + (\mathbf{n}_1, \mathbf{n}_4) + (\mathbf{n}_2, \mathbf{n}_3) + (\overline{\mathbf{n}}_1, \mathbf{n}_5) + (\overline{\mathbf{n}}_2, \mathbf{n}_6) + (\overline{\mathbf{n}}_3, \overline{\mathbf{n}}_6) + (\overline{\mathbf{n}}_4, \overline{\mathbf{n}}_5), \end{aligned}$$

$$(\overline{\mathbf{u}}_1; \overline{\mathbf{n}}_2) + (\mathbf{u}_1; \overline{\mathbf{n}}_3) + (\mathbf{u}_2; \overline{\mathbf{n}}_4) + (\mathbf{u}_3; \overline{\mathbf{n}}_5) + (\mathbf{u}_4; \overline{\mathbf{n}}_6) + (\mathbf{u}_5; \mathbf{n}_6) + u \to n\,. \tag{3.47}$$

By rewriting the traces above in terms of the group ranks we find that anomaly cancellation eqs. (3.42) constrain such ranks to satisfy

$$\begin{aligned} n_1 - n_4 &= u_1 - u_4\,, \\ n_1 + n_3 &= u_1 + u_3\,, \\ n_1 + n_2 &= u_1 + u_2\,, \\ n_2 + n_5 &= u_2 + u_5 = 4\,, \\ n_1 + n_3 + n_4 + n_6 &= u_1 + u_3 + u_4 + u_6 = 12\,. \end{aligned} \tag{3.48}$$

Thus we see that all u_j's may be expressed in terms of 55 integers, apart from u_1, which is a free integer parameter.

It is interesting to notice that if only eq. (3.42) are used then anomaly free (but not tadpole-free !) models, which have an asymmetric content in the 55 and 99 sector are possible.

For instance, a family of models obeying such constraint but not (3.45) is obtained by choosing

$$n_1 = n_4 = n_5 = 4\,, \tag{3.49}$$

$$n_2 = 0\,, \qquad n_3 = k\,, \qquad n_6 = 4-k\,, \tag{3.50}$$

$$u_1 = u_4 = u_5 = 0\,, \tag{3.51}$$

$$u_2 = 4\,, \qquad u_3 = k+4\,, \qquad u_6 = 8-k \tag{3.52}$$

with $k \leq 4$. The corresponding gauge groups are

$$\big[\mathrm{U}(4)\times\mathrm{U}(k+4)\times\mathrm{U}(8-k)\big]_{99}\times\big[\mathrm{U}(k)\times\mathrm{U}(4-k)\times\mathrm{U}(4)^3\big]_{55}\,. \tag{3.53}$$

These models are chiral and anomaly free, but are not symmetrical with respect to the exchange between D9 and D5 branes, and so violate (3.45).

In fact, eq. (3.45) reads $n_1 - n_2 + n_4 = u_1 - u_2 + u_4 = 4$ and thus imposes $u_j = n_j$ for all $j = 1, \ldots, 6$. With this extra condition the resulting gauge group is now

$$\begin{aligned}\big[\mathrm{U}(n_1)\times\mathrm{U}(n_2)\times\mathrm{U}(n_3)\times\mathrm{U}(4+n_2-n_1)\,\times \\ \times\,\mathrm{U}(4-n_2)\times\mathrm{U}(8-n_2-n_3)\big]_{55}\times[\mathrm{same}]_{99}\end{aligned} \tag{3.54}$$

symmetric under $5 \leftrightarrow 9$. These are the solutions which obey all tadpole cancellation conditions. For $n_1 = n_3 = 3$ and $n_2 = 2$ one recovers the solution given in [18].

4. Anomalous U(1)'s

It is known that orientifold compactifications lead to spectra which usually contain several Abelian factors with non-vanishing triangle anomalies. In [14] a generalized Green-Schwarz mechanism ensuring cancellation of such terms was found.[7] It involves RR scalars coming from the twisted closed sector of the string. In this section, we would like to indicate how U(1) anomalies actually cancel in the models we have considered above, whenever generic cancellation of non-Abelian anomalies is ensured.

[7] See [24, 20, 25] for the analogous mechanism in six dimensions.

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4.1 Mixed non-Abelian anomalies

For concreteness, let us consider the case of even orientifold models. The arguments below are also valid for odd orientifolds. In this subsection, we center on the mixed anomaly of the U(1) factor in U(n_i) in the Dp-brane sector with SU(n_j) non-Abelian group in the Dq-brane sector, denoted T_{ij}^{pq}.

Following [14] this triangle anomaly will be canceled if $T_{ij}^{pq} + A_{ij}^{pq} = 0$, where A_{ij}^{pq} is the Green-Schwarz term given by

$$A_{ij}^{pq} = \frac{1}{N} \sum_{k=1}^{N-1} C_k^{pq}(v) n_i^p \sin 2\pi k V_i^p \cos 2\pi k V_j^p \,. \tag{4.1}$$

Here k runs over twisted $\mathbb{Z}_N$ sectors, p, q run over 5,9 (meaning 5- or 9-brane origin of the gauge boson) and

$$C_k^{pp} = \prod_{a=1}^{3} 2 \sin \pi k v_a \quad \text{for } p = q \,, \qquad C_k^{59} = 2 \sin \pi k v_3 \,. \tag{4.2}$$

In principle, this factorization of U(1) anomalies could lead to new constraints on the spectrum of the model, beyond those imposed by generic cancellation of non-Abelian anomalies. In the following, we show this is *not* the case.

Sum over k in (4.1) can be performed explicitly. Consider the $p = q$ sector. By using (3.11) and orthogonality of cosines we find

$$A_{ij}^{pp} = -\frac{n_i}{2} \sum_{a=1}^{3} \left[\delta_{i,j+t_a} + \delta_{i,-j+t_a+1} - \delta_{i,-j-t_a+1} - \delta_{i,j-t_a}\right] \tag{4.3}$$

and similarly

$$A_{ij}^{59} = \frac{n_i}{2} \left[\delta_{i,j+(t_3/2)} + \delta_{i,-j+(t_3/2)+1} - \delta_{i,-j-(t_3/2)+1} - \delta_{i,j-(t_3/2)}\right] , \tag{4.4}$$

where arguments of the Kronecker δ functions are defined mod N. It is straightforward to check from the spectrum given in (3.21) and (3.22) that this indeed cancels the triangle anomaly whenever $i \neq j$.

The $i = j$ case is a little bit more involved due to contributions from antisymmetric representations. Recall that the contribution to the triangle anomaly from the antisymmetric $\mathbf{a}_{j\,(2)}$ of SU(n) is, in our normalization, $2((n-2)/2)$. Thus, the U(1)$_j$-SU(n_j) triangle anomaly reads

$$\begin{aligned} T_{jj}^{pp} = {} & \frac{n_j}{2}\Big[\delta_{j,(t_1+1)/2} + \delta_{j,(N+t_1+1)/2} - \delta_{j,(-t_1+1)/2} - \delta_{j,(N-t_1+1)/2} + (t_1 \to t_2)\Big] + \\ & + \frac{1}{2}\Bigg\{ \sum_{a=1}^{3} (n_{j-t_a} - n_{j+t_a}) + u_{j-t} - u_{j+t} - \\ & - 4\Big[\delta_{j,(t_1+1)/2} + \delta_{j,(N+t_1+1)/2} - \delta_{j,(-t_1+1)/2} + -\delta_{j,(N-t_1+1)/2}(t_1 \to t_2)\Big]\Bigg\}. \end{aligned} \tag{4.5}$$

The term between curly brackets is nothing but the expression (3.23) for cancellation of general non-Abelian anomalies. Since it must vanish the remaining contribution cancels the expression (4.3) for the case $i = j$.

This shows that the condition of cancellation of non-Abelian anomalies implies the appropriate factorization of U(1) anomalies. It is also easy to check that cubic U(1) anomalies are similarly canceled by the GS mechanism without the need of further constraints.

This behaviour could have been guessed, based on our results in section 3. There we noticed that the conditions of non-Abelian anomaly cancellation are equivalent to the tadpole cancellation conditions of a system of D3-branes at non-compact orientifold singularities (possibly in the presence of D7-branes). In this language, the additional tadpole conditions only arise in compact models, where the RR flux cannot escape to infinity. Thus the additional conditions are not required for consistency of the D3 brane system. Since the cancellation of U(1) anomalies is required for consistency even in the non-compact case, it cannot depend on any of the additional constraints. Thus, appropriate factorization of the U(1)'s must follow from the conditions already imposed by cancellation of non-Abelian anomalies, as we have shown above.

4.2 Mixed gravitational anomalies

We can use a similar computation to show that the cancellation of mixed U(1)-gravita-tional anomalies does not impose further constraints. This may appear puzzling at first sight, since gravitational anomalies are relevant only for compact models. One could expect their cancellation would involve some (or even all) of the additional tadpole cancellation conditions. So let us give an intuitive argument (of partial validity) to understand this fact before entering the detailed computation.

Again, we use the realization of these gauge theories in terms of D3-branes at orientifold singularities. Certainly, in the non-compact limit gravity propagates in all ten dimensions, not just in the four-dimensions where the gauge theory lives, and so there is no obvious reason why the mixed U(1)-gravitational anomalies should cancel. However, some of these singularities can be embedded in compact Calabi-Yau spaces (not necessarily toroidal orbifolds). This does not change the spectrum of the field theory on the D3-branes, since they only feel local physics, but has the effect that gravitational anomalies become relevant, and must cancel. Since the mixed U(1)-gravitational anomaly receives contributions only from fields living on the D3-branes, it follows that gravitational anomalies must cancel for non-compact singularities if they can be embedded in a global model. Following the argument in the previous subsection, this shows that, for these singularities, factorization of gravitational anomalies must be automatic once cancellation of non-Abelian anomalies is imposed.

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Obviously, this argument does not apply to general singularities, since most singularities cannot be embedded in global contexts. The explicit computation below, however, shows the conclusion concerning the absence of new constraints is indeed true for any singularity.

Let us consider the case of even order orientifolds (odd order orientifolds can be analyzed similarly). Starting from the spectrum given in section 3.2, the mixed anomaly $T_i^{\rm grav}$ for the ith U(1) factor (in the 55 sector) can be shown to be

$$\begin{aligned} T_i^{\rm grav} = & \sum_{a=1}^{3} (n_{i-t_a} n_i - n_i\, n_{i+t_a}) - n_i u_{i+t} + n_i u_{i-t} - \\ & - n_i \left[\delta_{i,(t_1+1)/2} + \delta_{i,(N+t_1+1)/2} - \delta_{i,(-t_1+1)/2} - \delta_{i,(N-t_1+1)/2} + (t_1 \to t_2)\right]. \end{aligned} \quad (4.6)$$

The gravitational anomaly for U(1) from the 99 sector has the same structure, with the replacement $n_i \leftrightarrow u_i$. For future convenience, let us note that by using the condition for the cancellation of non-Abelian anomalies (3.23), we can rewrite (4.6) as

$$T_i^{\rm grav} = 3n_i \left[\delta_{i,(t_1+1)/2} + \delta_{i,(N+t_1+1)/2} - \delta_{i,(-t_1+1)/2} - \delta_{i,(N-t_1+1)/2} + (t_1 \to t_2)\right]. \quad (4.7)$$

In [14] it was proposed that the GS contribution canceling this anomaly has the form

$$A_i^{\rm grav} = \frac{3}{4}\frac{1}{N}\sum_{k=1}^{N}\left[C_k^{55}(v)\,{\rm Tr}\,\gamma_{k,5} + C_k^{59}(v)\,{\rm Tr}\,\gamma_{k,9}\right] n_i \sin 2\pi k V_i\,. \quad (4.8)$$

This can be Fourier-transformed in a by now familiar way, and be expressed as

$$\begin{aligned} A_i^{\rm grav} = -\frac{3}{8}\, n_i \Bigg[& \sum_{a=1}^{3}(n_{i-t_a} + n_{-i+1+t_a} - n_{i+t_a} - n_{-i+1-t_a}) + \\ & + (u_{i-t} + u_{-i+1+t} - u_{i+t} + u_{-i+1-t})\Bigg]. \end{aligned} \quad (4.9)$$

Recalling the relations $n_i = n_{-i+1}$, $u_i = u_{-i+1}$, this reads

$$A_i^{\rm grav} = -\frac{3}{4}\, n_i \left[\sum_{a=1}^{3}(n_{i-t_a} - n_{i+t_a}) + (u_{i-t} - u_{i+t})\right], \quad (4.10)$$

which, after imposing the non-Abelian anomaly cancellation conditions (3.23), precisely cancels (4.7).

As an explicit example of the above discussion consider the $\mathbb{Z}_{12}$ orientifold. The GS factors eq. (4.8) are $A_1^{\rm grav} = -3n_1$, $A_3^{\rm grav} = 3n_3$, $A_4^{\rm grav} = -3n_4$ and $A_6^{\rm grav} = 3n_6$ while $A_2^{\rm grav} = A_5^{\rm grav} = 0$. The mixed gravitational anomalies T_i are easily computed from eq. (3.47). For instance for the first and second U(1) factors in the 55 sector we obtain

$$T_1^{\rm grav} = n_1[n_1 - n_2 + n_4 - 2n_5 + n_6 + u_3 - u_2 - 4] + 3n_1\,, \tag{4.11}$$
$$T_2^{\rm grav} = n_2[n_1 + n_3 - n_4 - n_5 - n_6 - u_1 + u_4]\,. \tag{4.12}$$

If non-Abelian anomaly cancellation eqs. (3.52) are used then terms inside brackets vanish and the correct contributions to be canceled by GS terms are obtained. The other cases proceed in the same way.

5. Conclusions and remarks

We have studied the relationship between cancellation of tadpoles and anomalies in compact type IIB $D = 4$ vacua. We have found that only a subset of the tadpole conditions are in general needed in order to get anomaly cancellations.

It is worth discussing what characterizes the twisted tadpoles whose cancellation is not required in order to get anomaly cancellation. In order to do that it is useful to do a T-duality transformation along the three complex compact dimensions. Now we have 3-branes and 7-branes (with their worldvolume including the first two complex compact planes) instead of 9-branes and 5-branes. We can now decompactify and consider the $D = 4$, $N = 1$ field theory living in the intersection of the 3-branes and 7-branes. Now, in this non-compact orientifold one has to impose the tadpole conditions corresponding to a given twist only if the flux of the RR charge originating at the fixed point cannot escape to infinity. For example, as we discussed in section 3, in the $\mathbb{Z}_3 \times \mathbb{Z}_3$ orientifold one has to impose the tadpole condition only for the twist $v = (1/3)(1, 1, -2)$, because for twists leaving one fixed complex plane like, e.g., $v = (1/3)(1, -1, 0)$ the RR flux can escape to infinity. In the $\mathbb{Z}_{12}$ case, which is another case in which a particular tadpole eq. (3.45) is not needed to get anomaly cancellation, something similar happens. In this case the twist is of the form $v = (1/4)(1, -1, 0)$ and the RR flux can escape to infinity (in the non-compact case) through the third complex plane which is transverse to the D7-brane worldvolume. Now, coming back to the *compact* orientifold case, since the massless charged spectrum from open strings is the same for both compact and non-compact cases, the models are still going to be anomaly-free. Thus the extra tadpole equations needed in the compact case will not be related to anomaly cancellation.

The $\mathbb{Z}_6{}'$ orientifold example shows again how things go. In this case, one would be tempted to say that tadpoles associated to the twist $2v = 1/3(1, 0, -1)$ should not be needed for anomaly cancellation, since the RR-flux could escape through the second complex plane. But that is not the case because the plane which is transverse to the D7-branes is the third one, which is rotated, no flux can escape through it.

From the $D = 4$ effective field theory point of view a natural question emerges. If there are tadpole constraints which are not needed for gauge anomaly cancellation, what is the low-energy symmetry which is guaranteed by them? We do not have a definite answer to that question but it seems sensible to believe that some other type of symmetries are guaranteed by them. In particular all these models have sigma-model U(1) symmetries as well as discrete gauge symmetries and cancellation of their anomalies could require the extra constraints [26].

Let us finally comment on the $D = 6$ case. In $D = 6$, $N = 1$ models, anomaly and tadpole cancellation conditions are totally equivalent since there are no twists with unrotated complex planes and no possibility for the RR flux to escape. Indeed, an explicit analysis along the lines we described for the $D = 4$ case gives rise to this equivalence. Concerning new solutions to the tadpole cancellation conditions, it is easy to see that the $\mathbb{Z}_2$, $\mathbb{Z}_3$, and $\mathbb{Z}_4$ $D = 6$, $N = 1$ models of [27, 28] have as unique solutions the ones given in those references. However the $\mathbb{Z}_6$ orientifold admits more solutions than those reported there. The reason for this, as it happened in the $D = 4$ case is that the $\mathbb{Z}_6$ twist (unlike the other three cases) has only one fixed point, the one at the origin. Thus the RR flux due to D9-branes can partially cancel that from D5-branes and lead to new solutions. The general group one can obtain is the same as that in (3.34).

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References

[1] M. Berkooz and R.G. Leigh, *A D=4 N=1 orbifold of type I strings*, *Nucl. Phys.* **B 483** (1997) 187 [hep-th/9605049].

[2] R.G. Leigh and M. Rozali, *Brane boxes, anomalies, bending and tadpoles*, *Phys. Rev.* **D 59** (1999) 026004 [hep-th/9807082].

[3] J. Blum and K. Intrilligator, *Consistency conditions for branes at orbifold singularities*, *Nucl. Phys.* **B 506** (1997) 223 [hep-th/9705030].

[4] J. Blum and K. Intrilligator, *New phases of string theory and 6d RG fixed points via branes at orbifold singularities*, *Nucl. Phys.* **B 506** (1997) 199 [hep-th/9705044].

[5] J. Park and A. Uranga, *A note on superconformal $N = 2$ theories and orientifolds*, *Nucl. Phys.* **B 542** (1999) 139 [hep-th/9808161].

[6] M.R. Douglas, B.R. Greene and D.R. Morrison, *Orbifold resolution by D-branes*, *Nucl. Phys.* **B 506** (1997) 84 [hep-th/9704151].

[7] S. Kachru, E. Silverstein, *4d conformal field theories and strings on orbifolds*, *Phys. Rev. Lett.* **80** (1998) 4855 [hep-th/9802183].

[8] A. Lawrence, N. Nekrasov and C. Vafa, *On conformal theories in four dimensions*, *Nucl. Phys.* **B 533** (1998) 199 [hep-th/9803015].

[9] A. Hanany and A. M. Uranga, *Brane boxes and branes on singularities*, *J. High Energy Phys.* **05** (1998) 013 [hep-th/9805139].

[10] A. Hanany and Y.-H. He, *Non-Abelian Finite Gauge Theories*, *J. High Energy Phys.* **9902** (1999) 013 [hep-th/9811183].

[11] J. Lykken, E. Poppitz and S. Trivedi, *Branes with GUTs and supersymmetry breaking*, *Nucl. Phys.* **B 543** (1999) 105 [hep-th/9806080].

[12] Z. Kakushadze, *On large N gauge theories from orientifolds*, *Phys. Rev.* **D 58** (1998) 106003 [hep-th/9804184].

[13] Z. Kakushadze, *Gauge theories from orientifolds and large N limit*, *Nucl. Phys.* **B 529** (1998) 157 [hep-th/9803214].

[14] L.E. Ibáñez, R. Rabadán and A.M. Uranga, *Anomalous* U(1)*'s in type I and type IIB* $D = 4$, $N = 1$ *string vacua*, *Nucl. Phys.* **B 542** (1999) 112 [hep-th/9808139].

[15] A. Sagnotti, in Cargese 87, *Strings on Orbifolds*, ed. G. Mack et al. (Pergamon Press, 1988) p. 521.

[16] G. Pradisi and A. Sagnotti, *Open string orbifolds*, *Phys. Lett.* **B 216** (1989) 59;
M. Bianchi and A. Sagnotti, *On the systematics of open string theories*, *Phys. Lett.* **B 247** (1990) 517.

[17] E. Gimon and J. Polchinski, *Consistency conditions for orientifolds and D-manifolds*, *Phys. Rev.* **D 54** (1996) 1667 [hep-th/9601038].

[18] G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, $D = 4$, $N = 1$, *type IIB orientifolds*, *Nucl. Phys.* **B 536** (1999) 29 [hep-th/9804026].

[19] L. Dixon, J.A. Harvey, C. Vafa and E. Witten, *Nucl. Phys.* **B 274** (1986) 285.

[20] M. Berkooz, R.G. Leigh, J. Polchinski, J.H. Schwarz, N. Seiberg and E. Witten, *Anomalies, dualities, and topology of* $D = 6N = 1$ *superstring vacua*, *Nucl. Phys.* **B 475** (1996) 115 [hep-th/9605184].

[21] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Ya.S. Stanev, *Chiral asymmetry in four-dimensional open-string vacua*, *Phys. Lett.* **B 385** (1996) 96 [hep-th/9606169].

[22] Z. Kakushadze and G. Shiu, *4-D Chiral $N = 1$ type I vacua with and without D5-branes*, *Nucl. Phys.* **B 520** (1998) 75 [hep-th/9706051]; *Phys. Rev.* **D 56** (1997) 3686 [hep-th/9705163].

[23] G. Zwart, *Four-dimensional $N = 1$ $\mathbb{Z}_N \times \mathbb{Z}_M$ Orientifolds*, *Nucl. Phys.* **B 526** (1998) 378 [hep-th/9708040].

[24] A. Sagnotti, *A note on the Green-Schwarz mechanism in open-string theories*, *Phys. Lett.* **B 294** (1992) 196 [hep-th/9210127].

[25] K. Intriligator, *RG fixed points in Six dimensions via branes at orbifold singularities*, *Nucl. Phys.* **B 496** (1997) 177 [hep-th/9702038].

[26] L.E. Ibáñez, R. Rabadán and A.M. Uranga, *Sigma-model anomalies in compact type IIB orientifolds and Fayet-Iliopoulos terms*, hep-th/9905098.

[27] E. Gimon and C. Johnson, *$K3$ Orientifolds*, *Nucl. Phys.* **B 477** (1996) 715 [hep-th/9604129].

[28] A. Dabholkar and J. Park, *Strings on Orientifolds*, *Nucl. Phys.* **B 477** (1996) 701 [hep-th/9604178].